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Modelling of heat transfer in an infiltrated granular bed in view of the difference of phase temperatures

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Abstract

Physically substantiated boundary conditions for problems of heat transfer in infiltrated granular beds based on the two-temperature model which allow for the absence of interphase heat transfer on boundaries are formulated. It is shown that classical Dankwerts conditions would be applicable for gas. The problem of porous cooling at the boundary conditions of the 2nd and 3rd kind on the outer boundary is solved in a new formulation.

There are a number of thermal processes in technology taking place in infiltrated granular beds, in which temperature drops between gas and particles have to be taken into account not treating a granular bed as a homogenous heat-conducting medium. First of all these are various non-stationary processes of granular bed heating/cooling by a flow of gas (liquid). Another example is the occurrence of temperature drops in motion of a heat flux toward gas when elements of the granular bed are produced from high-heat-conducting material (porous cooling of aircraft surfaces, blades of high-temperature gas turbines, etc.). For describing heat transfer in such cases the two-temperature model is used which in the simplest one-dimensional case has the form [1]:

$$c_f\rho_f\left(\varepsilon\frac{\partial T_f}{\partial t}+u\frac{\partial T_f}{\partial x}\right)=\lambda_f\varepsilon\frac{\partial^2 T_f}{\partial x^2}+\alpha_*(T_s-T_f) \quad (1)$$

$$c_s\rho_s(1-\varepsilon)\frac{\partial T_s}{\partial t}=\lambda_s(1-\varepsilon)\frac{\partial^2 T_s}{\partial x^2}+\alpha_*(T_f-T_s) \quad (2)$$

The coefficients λ_f and λ_s which are included in (1) and (2) and which determine the intensity of heat distribution over the phases are determined as [2]

$$\lambda_f=\lambda_f^0+\lambda_e, \quad (3)$$

where eddy conductivity of the gas phase [2] is

$$\lambda_e=0.03c_f\rho_f ud. \quad (4)$$

Thermal conductivity of an ensemble of particles (a frame of the bed) is expressed by the formula [2]

$$\lambda_s=\lambda_f^0(12+0.85\cdot Re\cdot Pr) \quad (5)$$

Note that thermal conductivity of the frame of the bed is mainly determined by heat conductivity of non-flow zones located near the points of contact between particles and it almost does not depend on heat conductivity of the particles. The volume coefficient of the interphase heat transfer is calculated as follows:

$$Nu_*=\frac{\alpha_* d^2}{\lambda_f^0}=NuS_{in}d, \quad (6)$$

where the coefficient of the interphase heat transfer α^* is determined by the formulas [3]

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Nomenclature

c_f, c_s	thermal conductivity of gas and particles, J/kg K
d	particle diameter, m
h	height of the granular bed, m
$Pe = c_f\rho_f u/\alpha_* h$;	
$Pe_f = c_f\rho_f uh/\lambda_f\varepsilon$;	
$Pe_s = c_f\rho_f uh/\lambda_s(1-\varepsilon)$ Peclet numbers	
$Pr = c_f\nu_f/\lambda_f^0$ Prandtl number	
q_0, q_1, q_2, q_w heat fluxes, W/m^2	
$Re = ud/\nu_f$ Reynolds number	
$Q_w = q_w/c_f\rho_f u(T^0 - T_0)$	
$St = \alpha/c_f\rho_f u$; $St_w = \alpha_w/c_f\rho_f u$ Stanton numbers	
S	cross-section of the bed, m^2
S_{in}	specific surface of particles (phase interface in the unit volume of the bed) in the case of packing of spheres $S_{in} = 6(1-\varepsilon)/d$
t	time, s
$t' = tu/h$	
T_f, T_s	temperature of gas and particles, K
T_0	inlet gas temperature, K
T_0'	gas temperature (at $x \to -0$), K
T^0	temperature of external incident flow, K
u	velocity of gas filtration rated at the full section of the bed, m/s
x	coordinate

Greek symbols

α_*	bulk coefficient of interphase heat transfer, W/m^3 K
α^*	coefficient of interphase heat transfer, W/m^2 K
α, α_w	coefficients of heat exchange between the granular bed and environment, W/m^2 K
ε	bed porosity
$\theta_f = (T_f - T_0)/(T^0 - T_0)$, $\theta_s = (T_s - T_0)/(T^0 - T_0)$	dimensionless temperatures
λ_f^0	molecular thermal conductivity of gas, W/m K
λ_f, λ_s	effective thermal conductivities of gas and solid phases, W/m K
ν_f	kinematic viscosity of gas, m^2/sec
$\xi = x/h$	dimensionless coordinate
ρ_f, ρ_s	densities of gas and particles, kg/m^3

Indices

e	vortex
f	gas
in	interphase
s	particles
w	surface (at $x = h$)

$$Nu = \frac{\alpha^* d}{\lambda_f^0} = 0.4\left(\frac{Re}{\varepsilon}\right)^{2/3} Pr^{1/3}, \quad \frac{Re}{\varepsilon} > 200 \tag{7}$$

$$Nu = 1.6 \times 10^{-2}\left(\frac{Re}{\varepsilon}\right)^{1.3} Pr^{1/3}, \quad \frac{Re}{\varepsilon} \leqslant 200 \tag{8}$$

As the analysis of examples of use of the system (1) and (2) for modelling of specific processes [2,4] has shown, the fundamental unresolved problem is a formulation of physically adequate boundary conditions. For this reason, in the present work the problem on formulation of correct boundary conditions is posed for Eqs. (1) and (2) based on the analysis of the interphase exchange process.

Consider the process of heat transfer, when gas (liquid) with temperature T_0 arrives at the disperse bed (Fig. 1).

(1) The boundary condition at $x = 0$ (gas)

Consider an elementary volume of the bed $dV = S dx$ near the boundary $x = 0$ (Fig. 2). The gas balance of heat fluxes in this volume is

$$(q_1 - q_0)S = \frac{\alpha_*}{S_{in}}(T_s - T_f)\,dS_{in}. \tag{9}$$

The value of the interphase surface in the considered volume is

$$dS_{in} = S_{in} dV = S_{in} S\, dx. \tag{10}$$

Based on Eqs. (10) and (9) can be written as

$$q_1 - q_0 = \alpha_*(T_s - T_f)\,dx. \tag{11}$$

From Eq. (11) the sought-for boundary condition obtained for $dx \to 0$ follows:

$$q_1 = q_0. \tag{12}$$

Based on $q_0 = c_f\rho_f u T_0'$ and $q_1 = c_f\rho_f u T_f(t,0) - \lambda_f\varepsilon\frac{\partial T_f(t,0)}{\partial x}$, this boundary condition finally takes the form

$$x = 0, \quad c_f\rho_f\frac{u}{\varepsilon}(T_f - T_0') = \lambda_f\frac{\partial T_f}{\partial x} \tag{13}$$

Condition (13) is defined as the well-known Dankwerts condition [5] used in modelling of transfer processes in the granular bed considered as a homogeneous medium. In this connection an important conclusion may be drawn in the context of the present work: in formulating the boundary conditions for the two-temperature model, phases on the bed boundaries can be

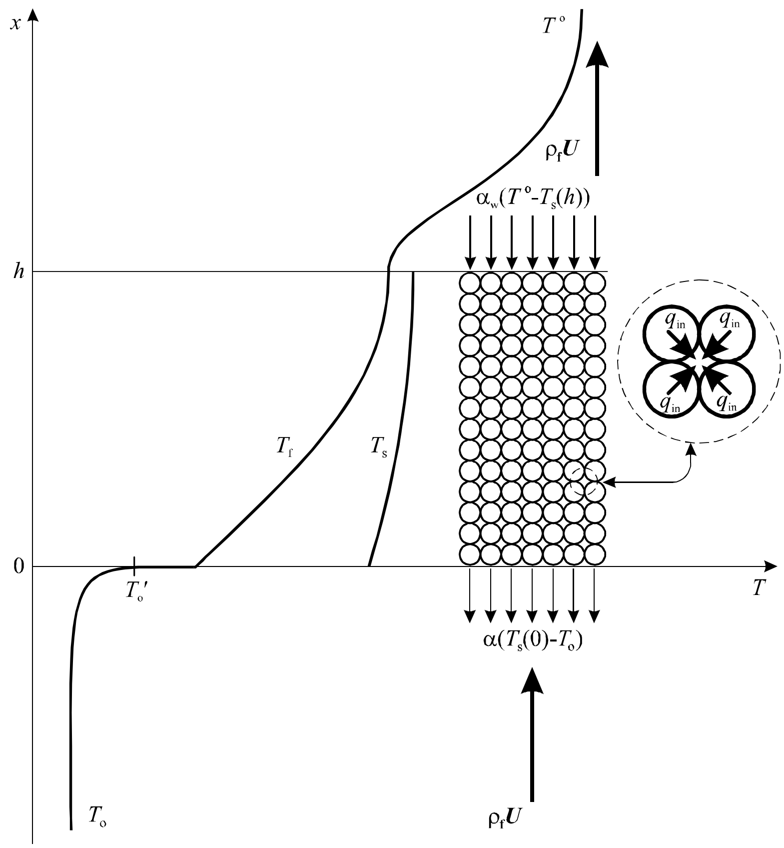

Fig. 1. The coordinate system, directions of heat and mass fluxes, and the character of distribution of phase temperatures inside the granular bed.

considered as isolated from each other because of the absence of interphase heat transfer.

(2) Boundary condition at $x = 0$ (particles)

In the absence of heat interaction of phases and in view of preliminary heat of gas (Fig. 1), the boundary condition takes the form:

$$x = 0, \quad (1 - \varepsilon)\lambda_s \frac{\partial T_s}{\partial x} = \alpha(T_s - T_0). \tag{14}$$

Note that with the use of $\lambda_s(1-\varepsilon)\frac{\partial T_s}{\partial x}\big|_{x=0} = c_f\rho_f u(T_0' - T_0)$, condition (13) is

$$x = 0, \quad c_f\rho_f u(T_f - T_0) = \varepsilon\lambda_f \frac{\partial T_f}{\partial x} + (1-\varepsilon)\lambda_s \frac{\partial T_s}{\partial x}. \tag{13a}$$

(3) The boundary condition at $x = h$ (gas)

In view of phase independence, the Dankwerts condition [5] can also be used here for the gas exit from the disperse medium:

$$x = h, \quad \frac{\partial T_f}{\partial x} = 0, \tag{15}$$

which testifies that the whole heat flux transferred by the gas is equal to convective one. Point that in [4] the hypothesis

$$x = h, \quad T_s = T_f \tag{15a}$$

was used which is not in agreement with (15).

(4) The boundary condition at $x = h$ (particles)

Consider three cases.

(a) Absence of an outer heat flux

This situation is common for non-stationary heat transfer. Probably, the sought-for condition has the form:

$$x = h, \quad \frac{\partial T_s}{\partial x} = 0. \tag{16}$$

Note that in [2] the equality

$$x = h, \quad \lambda_s \frac{\partial T_f}{\partial x} = \frac{\alpha_*}{S_{in}}(T_f - T_s) \tag{16a}$$

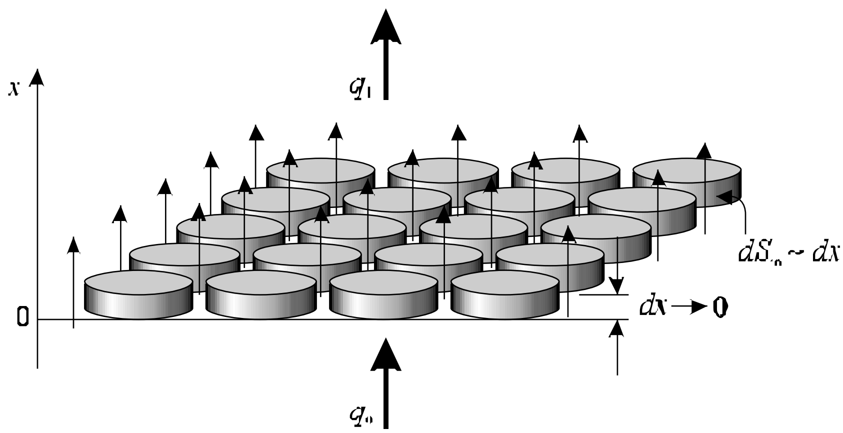

Fig. 2. Concerning the deduction of the boundary condition for gas at $x = 0$.

was used which, in the context of the present work, seems to be incorrect.

(b) The boundary condition of the 2nd kind

In this case, the outer heat flux q_2 is specified. Write the balance of heat fluxes through the considered boundary based on (15):

$$x = h, \quad -\lambda_s(1-\varepsilon)\frac{\partial T_s}{\partial x} + c_f\rho_f u T_f - c_f\rho_f u T^0 + q_2 = 0. \tag{17}$$

Note that in writing (17) it was presumed for simplicity that at the outlet the gas is heated up to the temperature of the environment T^0 (Fig. 1). In view of the so-called injection effect [6] $q_w = q_2 - c_f\rho_f u(T^0 - T(t,h))$, the unknown boundary condition

$$x = h, \quad \lambda_s(1-\varepsilon)\frac{\partial T_s}{\partial x} = q_w \tag{18}$$

is obtained. In [4], instead of (18) the condition

$$x = h, \quad \lambda_s(1-\varepsilon)\frac{\partial T_s}{\partial x} + \lambda_f\varepsilon\frac{\partial T_s}{\partial x} = q_w \tag{18a}$$

was used which transforms to (18) if condition (15) is satisfied.

(c) The boundary condition of the 3rd kind

On the basis of (18) unknown condition

$$x = h, \quad \lambda_s(1-\varepsilon)\frac{\partial T_s}{\partial x} = \alpha_w(T^0 - T_s(t,h)) \tag{19}$$

is obtained. Note that the coefficients α and α_w are calculated by the standard procedures [7].

Write the system (1), (2), (13)–(16), (18) and (19) in the dimensionless form:

$$\frac{\partial\theta_f}{\partial t'} + \frac{\partial\theta_f}{\partial\xi} = \frac{1}{Pe_f}\frac{\partial^2\theta_f}{\partial\xi^2} + \frac{1}{Pe}(\theta_s - \theta_f) \tag{20}$$

$$\frac{\partial\theta_s}{\partial t'} = \frac{1}{Pe_s}\frac{\partial^2\theta_s}{\partial\xi^2} + \frac{1}{Pe}(\theta_f - \theta_s). \tag{21}$$

The boundary-value conditions are

$$\theta_f(0,\xi) = \varphi(\xi); \quad \theta_s(0,\xi) = f(\xi) \tag{22}$$

$$\xi = 0, \quad \theta_f = \frac{1}{Pe_f}\frac{\partial\theta_f}{\partial\xi} + \frac{1}{Pe_s}\frac{\partial\theta_s}{\partial\xi} \tag{23}$$

$$\frac{\partial\theta_s}{\partial\xi} = St\cdot Pe_s\theta_s \tag{24}$$

$$\xi = 1, \quad \frac{\partial\theta_f}{\partial\xi} = 0. \tag{25}$$

(a) The boundary-value condition of the 2nd kind is

$$\frac{\partial\theta_s}{\partial\xi} = Pe_s\cdot Q_w. \tag{26}$$

In the absence of heat flux:

$$\frac{\partial\theta_s}{\partial\xi} = 0. \tag{26a}$$

(b) The boundary condition of the 3rd kind is

$$\frac{\partial\theta_s}{\partial\xi} = St_w\cdot Pe_s(1-\theta_s). \tag{27}$$

As follows from (20)–(27), the regularities of heat transfer in this system are determined by six similarity numbers: Pe, Pe_s, Pe_f, St, St_w, and Q_w.

Under the conditions of stationary heat transfer the obtained equations

$$\frac{d\theta_f}{d\xi} = \frac{1}{Pe_f}\frac{d^2\theta_f}{d\xi^2} + \frac{1}{Pe}(\theta_s - \theta_f) \tag{28}$$

$$0 = \frac{1}{Pe_s}\frac{d^2\theta_s}{d\xi^2} + \frac{1}{Pe}(\theta_f - \theta_s) \tag{29}$$

are simplified.

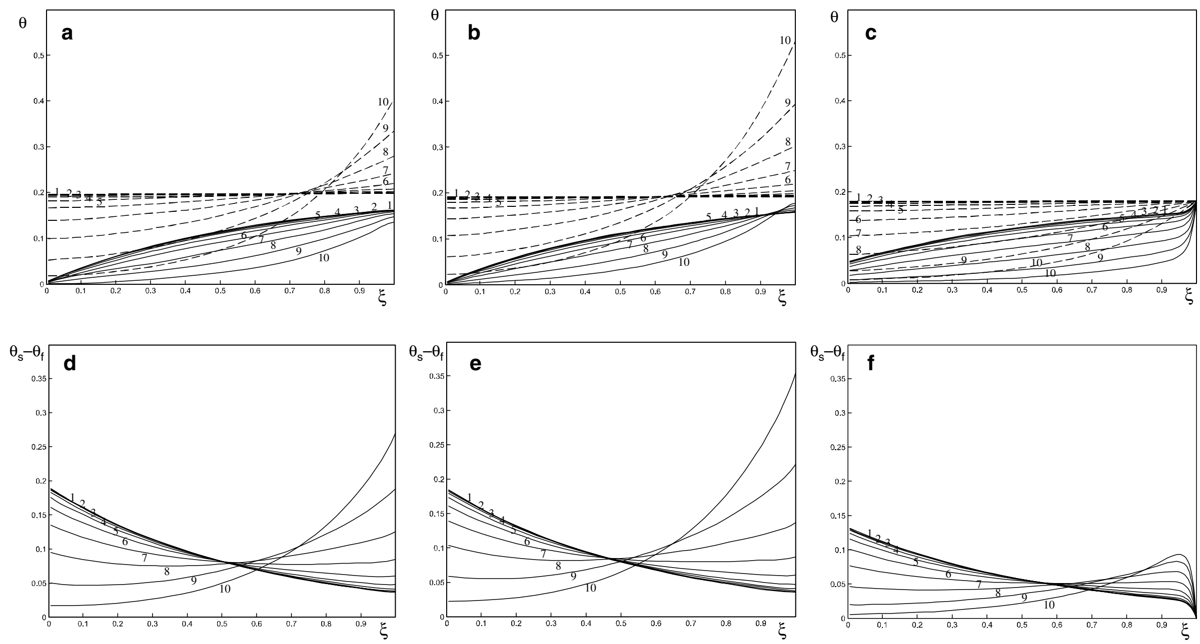

Fig. 3. Distributions of phase temperatures (a, b, c) and differences of phase temperatures (d, e, f) inside a thin granular bed. Solid lines—gas; dashed lines—particles. (1) $Pe_s = 0.01$; (2) 0.0215; (3) 0.0464; (4) 0.1; (5) 0.215; (6) 0.464; (7) 1; (8) 2.15; (9) 4.64; (10) 10. (a) and (d) boundary conditions of the 3rd kind; (b) and (e) boundary conditions of the 2nd kind; (c) and (f) calculation by the model of [4].

The boundary conditions are

$$\xi = 0, \quad \theta_f = \frac{1}{Pe_f}\frac{d\theta_f}{d\xi} + \frac{1}{Pe_s}\frac{d\theta_s}{d\xi} \tag{30}$$

$$\frac{d\theta_s}{d\xi} = St \cdot Pe_s \theta_s \tag{31}$$

$$\xi = 1, \quad \frac{d\theta_f}{d\xi} = 0 \tag{32}$$

The boundary condition of the 2nd kind is

$$\frac{d\theta_s}{d\xi} = Pe_s \cdot Q_w, \tag{33}$$

The boundary condition of the 3rd kind is

$$\frac{d\theta_s}{d\xi} = St_w \cdot Pe_s(1 - \theta_s). \tag{34}$$

Note that the system (28)–(34) is the general determination of the important applied problem of porous cooling [6].

Figs. 3 and 4 show the results of the numerical solution of problem (28)–(34). The distributions θ_f, θ_s, and $\theta_s - \theta_f$ are obtained for various Pe_s numbers and two values of h/d:$h/d = 6.7$ (a thin bed) and $h/d = 67$ (a thick bed). The initial values of the dimensionless parameters are given in Table 1.

Figs. 3c, f, and 4c, f give the results of calculations by (28) and (29) with the boundary conditions adopted in [4]:

$$\xi = 0, \quad \frac{1}{Pe_s}\frac{d\theta_s}{d\xi} + \frac{1}{Pe_f}\frac{d\theta_f}{d\xi} = \theta_f, \tag{35}$$

$$\frac{d\theta_s}{d\xi} = St \cdot Pe_s \theta_s; \tag{36}$$

$$\xi = 1, \quad \theta_f = \theta_s, \tag{37}$$

$$\frac{1}{Pe_s}\frac{d\theta_s}{d\xi} + \frac{1}{Pe_f}\frac{d\theta_f}{d\xi} = Q_w. \tag{38}$$

As it is shown in Figs. 3 and 4, the distributions θ_f, θ_s, and $\theta_s - \theta_f$ almost have no difference for the boundary conditions of the 2nd and 3rd kinds. The Pe_s number dependence (heat conduction λ_s) is quite significant in all cases. It is important to note that the condition of the equality of phase temperatures at the outlet adopted in [4] greatly distorts the temperature fields, especially at large values of Pe_s (small λ_s). The character of the distributions $\theta_s - \theta_f$ shown in Figs. 3f and 4f clearly denotes the artificiality of the assumption that $T_s = T_f$ at $x = h$.

Integral consideration of characteristic features of the heat regime inside the disperse bed can easily be made on the basis of the equation

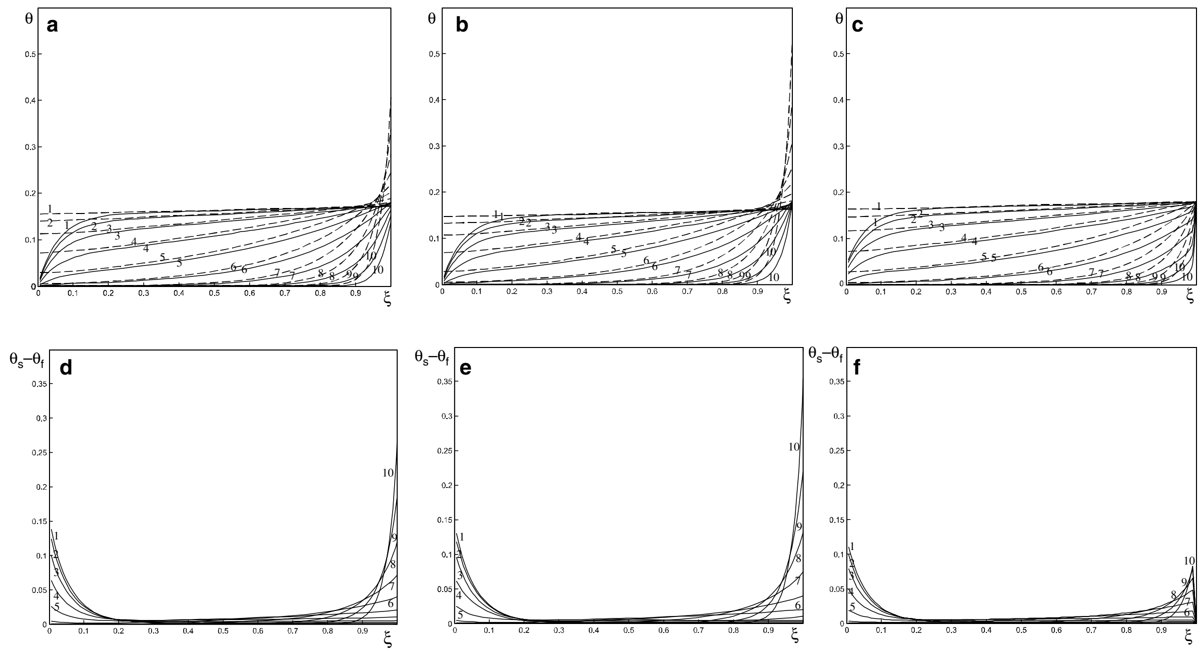

Fig. 4. Distributions of phase temperatures (a, b, c) and differences of phase temperatures (d, e, f) inside the thick granular bed. Solid lines—gas; dashed lines—particles. (1) $Pe_s = 0.1$; (2) 0.215; (3) 0.464; (4) 1; (5) 2.15; (6) 4.64; (7) 10; (8) 21.5; (9) 46.4; (10) 100. (a)–(f) see Fig. 3.

Table 1
Dimensionless parameters used in calculations

Parameter	Thin bed	Thick bed
St	0.24	0.24
St_w	0.23	0.23
Q_w	0.18	0.18
Pe	0.57	0.057
Pe_f	63.2	632
Pe_s	0.01–10	0.1–100

$$c_f\rho_f u(T_f(h) - T'_0) = \int_0^h \alpha_*(T_s - T_f)\,dx, \tag{39}$$

which is obtained due to integration of (1) with account for the Dankwerst conditions (13) and (15). With the use of (39) the general equation of heat balance in the bed for the boundary condition of the 2nd kind takes the form

$$q_w = c_f\rho_f u(T_f(h) - T'_0) + \alpha(T_s(0) - T_0) \tag{40}$$

The quantity $\Psi_T = \int_0^h \alpha_*(T_s - T_f)\,dx/q_w$, which in [4] was given the name of the parameter of porous cooling efficiency, on the basis of (39) and (40) becomes

$$\Psi_T = 1 - \frac{\alpha(T_s(0) - T_0)}{q_w} \tag{41}$$

The expression Ψ_T for boundary conditions of the 3rd kind is similar. In Fig. 5, calculations of Ψ_T for both systems of the boundary conditions are given. As is seen, values of Ψ_T and the character of the dependence on the Pe_s number substantially differ for the boundary conditions of [4] and for those adopted in the present work. It is interesting to note that if the hypothesis of the equality of the phase temperatures at the outlet from the bed practically does not distort the temperature fields of phases at large values of Pe_s (Figs. 3 and 4), the quantity Ψ_T turns out to be more sensitive even at large values of Pe_s. It should be noted that the hypothesis on $T_s = T_f$ at $x = h$ adopted in [4] leads to the underestimated values of $T_s - T_f$ and, as a consequence, to the lower values of the efficiency parameter Ψ_T (Fig. 5).

As a result of the present work, the physically justified boundary conditions (13)–(15), (18) and (19), which are based on the account of the fact that the phases on the boundaries are isolated, are formulated. The dependence of the quantities T_f, T_s, $T_s - T_f$, and Ψ_T on thermal conductivity of the frame of the bed and its height are revealed. It is shown that the hypothesis on the equality of phase temperatures at the outlet from the bed adopted in [4] has a limited area of application and can be used justifiably only at small values of Pe_s (large values of thermal conductivity of the frame of the bed).

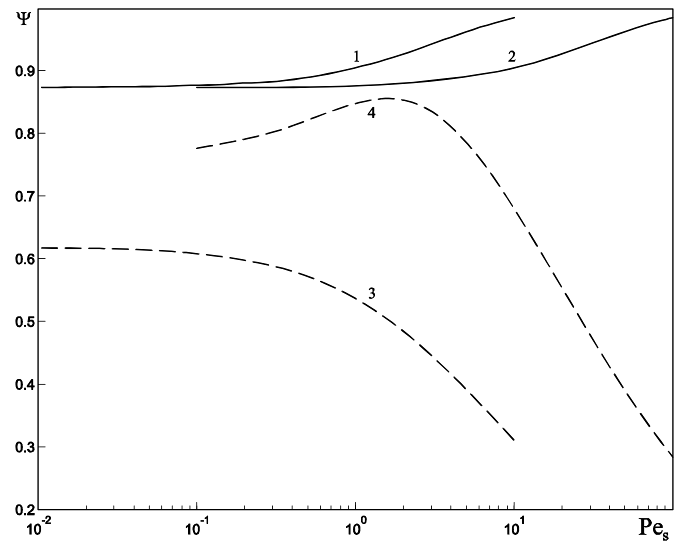

Fig. 5. Pe_s dependence of the efficiency parameter Ψ_T; (1) boundary conditions of the 2nd and 3rd kinds for a thin granular bed; (2) boundary conditions of the 2nd and 3rd kinds for a thick granular bed; (3) the model of [4] for a thin granular bed and (4) the model of [4] for a thick granular bed.

References

[1] M.E. Aerov, O.M. Todes, D.A. Narinskii, Apparatus with a Stationary Granular Bed, Khimiya, Leningrad, 1979.

[2] Yu.Sh. Matros, V.I. Lugovskii, V.L. Ogarkov, V.B. Nakrokhin, Heat transfer in a blow-through bed in a motionless granular bed, Teor. Osnovy Khim. Tekhnol. 12 (1978) 291–293.

[3] N.I. Gelperin, V.G. Ainshtein, V.B. Kvasha, Fundamentals of Fluidization Technique, Khimiya, Moscow, 1967.

[4] Yu.V. Polezhaev, E.M. Seliverstov, Universal model of heat transfer in the systems with penetrating cooling, Teplofiz. Vys. Temp. 40 (2002) 922–930.

[5] V.A. Borodulya, Yu.P. Gupalo, Mathematical Models of Chemical Reactors with Fluidized Bed, Nauka i Tekhnika, Minsk, 1976.

[6] Yu.V. Polezhaev, F.B. Yurevich, Thermal Protection, Nauka i Tekhnika, Minsk, 1976.

[7] V.P. Isachenko, V.A. Osipova, A.S. Sukomel, Heat Transfer, Energoizdat, Moscow, 1981.